2 Insurance and Annuity

Let's Learn

- Fire, Marine and Accident Insurance
- Annuity
  - Terminology of Annuity
  - Annuity Due
  - Sinking Fund

Introduction

Life is full of risk. Risk is due to uncertainty. It involves a loss or some other undesirable or negative outcome. All of us face some type of risk in every-thing we do and every decision we make. We often look for ways to avoid risk by taking steps to prevent it. However, it is impossible to completely prevent every possible risk. As an alternative, we try to minimize the impact of risk by having insurance or some other type of protection from loss. Insurance is a way of managing the risk in order to protect our life, property, vehicle, or other financial assets against possible loss or damage due to contingencies like burglary, fire, flood, earthquake, etc.

The verb "to insure" means to arrange for compensation in the event of damage or total loss of property or injury or the death of someone, in exchange of regular payments to a company or to the state. The word "insurance" means creation of some security or monetary protection against a possible damage or loss. Insurance is a legal contract between an insurance company (insurer) and a person covered by the insurance (insured). An insurance policy is a legal document of the contract or agreement between the two parties, the insured and the insurer.

Insurance is of two types: Life Insurance and General Insurance.

(1) Life insurance

A person who wishes to be insured for life agrees to pay the insurance company a certain amount of money periodically. This amount is called the premium. The period of the payment can be a month, a quarter, half-year, or a year. In return, the insurance company agrees to pay a definite amount of money in the event of death of the insured or maturity of the policy, that is, at end of the contract period. This amount is called the policy value.

(2) General Insurance

General insurance covers all types of insurance except life insurance. General insurance allows a person to insure properties like buildings, factories, and goodowns containing goods against a possible loss (total or partial) due to fire, flood, earthquake, etc.

Vehicles can be insured to cover the risk of possible damage due to accidents.

In case of loss or damage, the insurance company pays compensation in the same proportion that exists between the policy value and the property value.

All contracts of general insurance are governed by the principle of indemnity, which states that an insured may not be compensated by the insurance company in an amount exceeding the insured's economic loss. As a result, an insured person cannot make profit from an insurance policy.

Let's Learn

2.1 Fire, Marine, and Accident Insurance

(1) Fire Insurance

Fire insurance is property insurance that covers damage and losses caused by fire to property like buildings, godowns containing goods, factories, etc. It is possible to insure the entire property or only its part. The value of the property is called Property Value. The value of the insured part of property is called Policy Value. The amount paid to the insurance company to insure the property is called premium.

Primium = Rate of Premium × Policy Value

The period of a fire insurance policy is one year and the premium is expressed as percentage of the value of the insured property.

In case of damage to the property due to fire, the insurance company agrees to pay compensation in the proportion that exist between policy value and property value. The value of the damage is called "loss" and the amount that the insured can demand under the policy is called claim.

$$\therefore \quad \text{Claim} = \text{Loss} \times \frac{\text{PolicyValue}}{\text{PropertyValue}}$$

(2) Accident Insurance

Personal accident insurance is a policy that can reimburse your medical costs, provide compensation in case of disability or death caused by accidents. Accident insurance allows insuring vehicles like cars, trucks, two wheelers, etc. against to a vehicle due to accidents. This policy also covers the liability of the insured person to third parties involved in the accident. The period of such policies is one year.

(3) Marine Insurance

Marine Insurance covers goods, freight, cargo, etc. against loss or damage during transit by road, rail, sea or air. Shipments are protected from the time they leave the seller's warehouse till the time they reach the buyer's warehouse. Marine insurance offers complete protection during transit goods and compensates in the events of any loss.

The party responsible for insuring the goods is determined by the sales contract. The amount of claim is calculated by the same method that is used in the case of fire insurance.

SOLVED EXAMPLES

Ex. 1: A building worth Rs.50,00,000 is insured for $\left(\frac{4}{5}\right)^{th}$ of its value at a premium of 5%. Find the amount of premium. Also, find commission of the agent if the rate of commission is 3%.

Solution:

Property value = Rs. 5000000

Policy value $= \frac{4}{5} \times 5000000$

= Rs. 4000000

Rate of premium = 5%

$\therefore$ Amount of premium $= 4000000 \times \frac{5}{100}$

$\therefore$ Premium amount = Rs. 200000

$\therefore$ Commission at 3% $= 200000 \times \frac{3}{100}$

= Rs.6000

$\therefore$ Agent's commission = Rs.6000

Ex. 2: A shopkeeper insures his shop valued Rs. 20 lakh for 80% of its value. He pays a premium of Rs.80000. Find the rate of premium. If the agent gets commission at 12%, find the agent's commission.

Solution:

Property value = Rs. 2000000

Insured value = 80% of property value

$= 2000000 \times \frac{80}{100}$

= Rs. 1600000

Now, the premium paid = Rs.80000

$\therefore$ Rate of premium $= \dfrac{100 \times 80000}{1600000}$

$\therefore$ Rate of premium = 5%

Commission paid at 12% of premium

$= 80000 \times \dfrac{12}{100}$

= Rs. 9600.

$\therefore$ Agent's commission is Rs.9600.

Ex. 3: A car worth Rs.5,40,000 is insured for Rs.4,50,000. The car is damaged to the extent of Rs.2,40,000 is an accident. Find the amount of compensation that can be claimed under the policy.

Solution:

Value of the car = Rs. 540000

Insured value = Rs. 450000

Damage = Rs. 240000

$\text{Claim} = \dfrac{\text{Insured value}}{\text{Property value}} \times \text{loss}$

$= \dfrac{450000}{540000} \times 240000$

= Rs.200000

$\therefore$ A compensation of Rs. 2,00,000 can be claimed under the policy.

Ex. 4: 10000 copies of a book, priced Rs. 80 each were insured for $\left(\dfrac{3}{5}\right)^{th}$ of their value. Some copies of the book were damaged in transit, and were therefore reduced to 60% of their value. If the amount recovered against the damage was Rs.24000, find the number of damaged copies of the book.

Solution :

Total number of copies = Rs. 10000

Cost of one book = Rs.80

$\text{Insured value} = \dfrac{3}{5} \times \text{Property value}$

Insurance claim = Rs.24000

$\text{Now, claim} = \dfrac{\text{Insured value}}{\text{Property value}} \times \text{loss}$

$\therefore \quad 24000 = \dfrac{3}{5} \times \text{loss}$

$\therefore \quad \text{loss} = 24000 \times \dfrac{5}{3}$

= Rs.40000

This amount was equal to 40% of the damage.

$\therefore$ `Total damage $= 40000 \times \dfrac{100}{40}$

= Rs.100000

Since cost of one book was Rs. 80

The number of books damaged $= \dfrac{100000}{80}$

$\therefore$ 1250 books were damaged.

Ex. 5: A cargo valued at Rs.10,00,000 was insured for Rs.7,00,000 during a voyage. If the rate of premium is 0.4%. find (i) the amount of premium, (ii) The amount that can be claimed if the cargo worth Rs.6,00,000 is destroyed, (iii) the amount that can be claimed, if cargo worth Rs.6,00,000 is destroyed completely and the remaining cargo is so damaged that its value is reduced by 40%.

Solution:

Property value = Rs. 10,00,000

Policy value = Rs. 7,00,000

Rate of premium = 0.4%

i) Premium = 0.4% of policy value

$= 700000 \times \dfrac{0.4}{100}$

= Rs. 2800

$\therefore$ Total Premium = Rs. 2800

ii) Claim $= \text{loss} \times \dfrac{\text{PolicyValue}}{\text{PropertyValue}}$

$= 600000 \times \dfrac{700000}{1000000}$

= Rs. 420000

iii) Total value of cargo = Rs. 1000000

Value of the cargo completely destroyed = Rs.600000

∴ Value of remaining cargo = Rs.400000

Loss on value of remaining cargo = 40% of the value of remaining cargo

$$= \frac{40}{100} \times 400000$$

$$= \text{Rs.}160000$$

∴ Total loss = 600000 + 160000

= Rs.760000

$$\therefore \quad \text{Claim} = \text{loss} \times \frac{\text{PolicyValue}}{\text{PropertyValue}}$$

$$= 760000 \times \frac{700000}{1000000}$$

= Rs. 532000

Ex. 6: A property worth Rs.4,00,000 is insured with three companies X, Y and Z for amounts Rs.1,20,000, Rs.80,000, and Rs. 1,00,000 respectively. A fire caused a loss of Rs. 2,40,000, Calculate the amounts that can be claimed from the three companies.

Solution:

Loss = Rs.2,40,000

$$\text{Claim} = \text{Loss} \times \frac{\text{PolicyValue}}{\text{PropertyValue}}$$

$$\text{Claim from company X} = 2{,}40{,}000 \times \frac{120000}{400000}$$

= Rs. 72,000

$$\text{Claim from company Y} = 2{,}40{,}000 \times \frac{80000}{400000}$$

= Rs. 48,000

$$\text{Claim from company Z} = 2{,}40{,}000 \times \frac{100000}{400000}$$

= Rs. 60,000

Ex. 7: An agent places insurance for Rs. 4,00,000 on life of a person. The premium is to be paid annually at the rate of Rs.35 per thousand per annum. Find the agent's commission at 15% on the first premium.

Policy value = Rs. 4,00,000

Rate of premium = Rs.35 per thousand p.a.

$$\therefore \text{Amount of premium} = \frac{35}{1000} \times 4{,}00{,}000$$

= Rs.14,000

Rate of commission = 15%

$$\therefore \text{Amount of commission} = 14000 \times \frac{15}{100}$$

= Rs.2100

Ex. 8: A person takes a life policy of Rs. 2,00,000 for 15 years. The rate of premium is Rs. 55 per thousand per annum. If the bonus is paid at the average rate of Rs. 6 per thousand, what is the benefit to the insured?

Policy value = Rs. 2,00,000

Rate of premium = Rs.55 per thousand p.a.

$$\therefore \text{Amount of premium} = \frac{55}{1000} \times 2{,}00{,}000$$

= Rs.11,000

The insured pays premium for 15 years.

∴ Total premium paid = 11000 × 15

= Rs.165000

Rate of bonus is Rs.6 per thousand per annum on the policy value. Therefore, on the policy Rs. 2,00,000

bonus for 1 year = 6 × 200

= Rs.1200

∴ bonus for 15 year = 1200 × 15

= Rs.18000

Hence, when the policy matures,

the insured gets = 2,00,000 + 18000

= Rs.2,18,000

∴ Benefit = 218000 − 165000

= Rs.53000

EXERCISE 2.1

1. Find the premium on a property worth Rs. 25,00,000 at 3% if (i) the property is fully insured, (ii) the property is insured for 80% of its value.

2. A shop is valued at Rs. 3,60,000 for 75% of its value. If the rate of premium is 0.9%, find the premium paid by the owner of the shop. Also, find the agents commission if the agent gets commission at 15% of the premium.

3. A person insures his office valued at Rs. 5,00,000 for 80% of its value. Find the rate of premium if he pays Rs.13,000 as premium. Also. find agent's commission at 11%

4. A building is insured for 75% of its value. The annual premium at 0.70 per cent amounts to Rs.2625. If the building is damaged to the extent of 60% due to fire, how much can be claimed under the policy?

5. A stock worth Rs.7,00,000 was insured for Rs.4,50,000. Fire burnt stock worth Rs.3,00,000 completely and damaged the remaining stock to the extent of 75 % of its value. What amount can be claimed under the policy?

6. A cargo of rice was insured at 0.625 % to cover 80% of its value. The premium paid was Rs.5250. 1f the price of rice is Rs.21 per kg. find the quantity of rice (in kg) in the cargo.

7. 60,000 articles costing Rs.200 per dozen were insured against fire for Rs. 2,40,000. If 20% of the articles were burnt and 7200 of the remaining articles were damaged to the extent of 80% of their value, find the amount that can be claimed under the policy.

8. The rate of premium is 2% and other expenses are 0.75%. A cargo worth Rs.3,50,100 is to be insured so that all its value and the cost of insurance will be recovered in the event of total loss.

9. A property worth Rs. 4.00.000 is insured with three companies. A. B. and C. The amounts insured with these companies are Rs.1,60,000, Rs.1,00,000 and Rs.1,40,000 respectively. Find the amount recoverable from each company in the event of a loss to the extent of Rs. 9.000.

10. A car valued at Rs.8,00,000 is insured for Rs.5,00,000. The rate of premium is 5% less 20%. How much will the owner bear including the premium if value of the car is reduced to 60 % of its original value.

11. A shop and a godown worth Rs.1,00,000 and Rs.2,00,000 respectively were insured through an agent who was paid 12% of the total premium. If the shop was insured for 80% and the godown for 60% of their respective values, find the agent's commission, given that the rate of premium was 0.80% less 20% .

12. The rate of premium on a policy of Rs. 1,00,000 is Rs.56 per thousand per annum. A rebate of Rs.0.75 per thousand is permitted if the premium is paid annually. Find the net amount of premium payable if the policy holder pays the premium annually.

13. A warehouse valued at Rs.40,000 contains goods worth Rs.240,000. The warehouse is insured against fire for Rs.16,000 and the goods to the extent of 90% of their value. Goods worth Rs.80,000 are completely destroyed, while the remaining goods are destroyed to 80% of their value due to a fire. The damage to the warehouse is to the extent of Rs.8,000. Find the total amount that can be claimed.

14. A person takes a life policy for Rs.2,00,000 for a period of 20 years. He pays premium for 10 years during which bonus was declared at an average rate of Rs.20 per year per thousand. Find the paid up value of the policy if he discontinues paying premium after 10 years.

Let's Study

2.2 Annuity

When you deposit some money in a bank, you are entitled to receive more money (in the form of interest) from the bank than you deposit, after a certain period of time. Similarly, when people borrow money for a certain period of time, they pay back more money (again, in the form of interest). These two examples show how money has a time value. A rupee today is worth more than a rupee after one year. The time value of money explains why interest is paid or earned. Interest, whether it is on a bank deposit or a loan, compensates the depositor or lender for the time value of money. When financial transactions occur at different points of time, they must be brought to a common point in time to make them comparable.

Consider the following situation. Ashok deposits Rs.1000 every year in his bank account for 5 years at a compound interest rate of 10 per cent per annum. What amount will Ashok receive at the end of five years? In other words, we wish to know the future value of the money Ashok deposited annually for five years in his bank account.

Assuming that Rs.1000 are deposited at end of every year, the future value is given by

$$1000\left[\left(1+\frac{1}{10}\right)^4+\left(1+\frac{1}{10}\right)^3+\left(1+\frac{1}{10}\right)^2+\left(1+\frac{1}{10}\right)+1\right]$$

$= 6105.1$

In the same situation, the present value of the amount that Ashok deposits in his bank account is given by

$$\frac{1000}{(1.10)^1}+\frac{1000}{(1.10)^2}+\frac{1000}{(1.10)^3}+\frac{1000}{(1.10)^4}+\frac{1000}{(1.10)^5}$$

$=$ Rs.3790.78

We also come across a situation where a financial company offers to pay Rs.8,000 after 12 years for Rs.1000 deposited today. In such situations, we wish to know the interest rate offered by the company.

Studies of this nature can be carried out by studying Annuity.

An annuity is a sequence of payments of equal amounts with a fixed frequency. The term "annuity" originally referred to annual payments (hence the name), but it is now also used for payments with any frequency. Annuities appear in many situations: for example, interest payments on an investment can be considered as an annuity. An important application is the schedule of payments to pay off a loan. The word "annuity" refers in everyday language usually to a life annuity. A life annuity pays out an income at regular intervals until you die. Thus, the number of payments that a life annuity makes is not known. An annuity with a fixed number of payments is called an annuity certain, while an annuity whose number of payments depend on some other event (such as a life annuity) is a contingent annuity. Valuing contingent annuities requires the use of probabilities.

Definition

An annuity is a series of payments at fixed intervals, guaranteed for a fixed number of years or the lifetime of one or more individuals. Similar to a pension, the money is paid out of an investment contract under which the annuitant(s) deposit certain sums (in a lump sum or in installments) with an annuity guarantor (usually a government agency or an insurance company). The amount paid back includes principal and interest.

1.2.1 Terminology of Annuity

Four parties to an annuity

Annuitant - A person who receives an annuity is called the annuitant.

Issuer - A company (usually an insurance company) that issues an annuity.

Owner - An individual or an entity that buys an annuity from the issuer of the annuity and makes contributions to the annuity.

Beneficiary - A person who receives a death benefit from an annuity at the death of the annuitant.

Two phases of an annuity

Accumulation phase - The accumulation (or investment) phase is the time period when money is added to the annuity. An annuity can be purchased in one single lump sum (known as a single premium annuity) or by making investments periodically over time.

Distribution phase - The distribution phase is when the annuitant receiving distributions from the annuity. There are two options for receiving distributions from an annuity. The first option is to withdraw some or all of the money in the annuity in lump sums. The second option (commonly known as guaranteed income or annuitization option) provides a guaranteed income for a specific period of time or the entire lifetime of the annuitant.

Types of Annuities - There are three types of annuities.

(i) **Annuity Certain.** An annuity certain is an investment that provides a series of payments for a set period of time to a person or to the person's beneficiary. It is an investment in retirement income offered by insurance companies. The annuity may also be taken as a lump sum.

Because it has a set expiration date, a annuity certain generally pays a higher rate of return than lifetime annuity. Typical terms are 10, 15, or 20 years.

Contingent Annuity. Contingent annuity is a form of annuity contract that provides payments at the time when the named contingency occurs. For instance, upon death of one spouse, the surviving spouse will begin to receive monthly payments. In a contingent annuity policy the payment will not be made to the annuitant or the beneficiary until a certain stated event occurs.

Perpetual Annuity or Perpetuity. A perpetual annuity, also called a perpetuity promises to pay a certain amount of money to its owner forever.

Though a perpetuity may promise to pay you forever, its value isn't infinite. The bulk of the value of a perpetuity comes from the payments that you receive in the near future, rather than those you might receive 100 or even 200 years from now.

Classification of Annuities - Annuities are classified in three categories according to the commencement of income. These three categories are: Immediate Annuity, Annuity Due, and Deferred Annuity.

Immediate Annuity or Ordinary Annuity - The immediate annuity commences immediately after the end of the first income period. For instance, if the annuity is to be paid annually, then the first installment will be paid at the expiry of one year. Similarly in a half- yearly annuity, the payment will begin at the end of six months. The annuity can be paid either yearly, half-yearly, quarterly or monthly.

The purchase money (or consideration) is in a single amount. Evidence of age is always asked for at the time of entry.

Annuity Due - Under this annuity, the payment of installment starts from the time of contract. The first payment is made as soon as the contract is finalized. The premium is generally paid in single amount but can be paid in installments as is discussed in the deferred annuity. The difference between the annuity due and immediate annuity is that the payment for each period is paid in its beginning under the annuity due contract while at the end of the period in the immediate annuity contract.

Deferred Annuity - In this annuity contract the payment of annuity starts after a deferment period or at the attainment by the annuitant of a specified age. The premium may be paid as a single premium or in instalments.

The premium is paid until the date of commencement of the instalments.

We shall study only immediate annuity and annuity due.

Present value of an annuity - The present value of an annuity is the current value of future payments from an annuity, given a specified rate of return or discount rate. The annuity's future cash flows are discounted at the discount rate. Thus the higher the discount rate, the lower the present value of the annuity.

Future value of an annuity - The future value of an annuity represents the amount of money that will be accumulated by making consistent investments over a set period, assuming compound interest. Rather than planning for a guaranteed amount of income in the future by calculating how much must be invested now, this formula estimates the growth of savings given a fixed rate of investment for a given amount of time.

The present value of an annuity is the sum that must be invested now to guarantee a desired payment in the future, while the future value of an annuity is the amount to which current investments will grow over time.

Note:

(1) We consider only uniform and certain annuities.

(2) If the type of an annuity is not mentioned, we assume that the annuity is immediate annuity.

(3) If there is no mention of the type of interest, then it is assumed that the interest is compounded per annum.

If payments are made half-yearly (that is, twice per year), then r is replaced by $\frac{r}{2}$ (the compounding rate) and n is replaced by $2n$ (the number of time periods).

If payments are made quarterly (that is, four times per year), then r is replaced by $\frac{r}{4}$ (the compounding rate) and n is replaced by $4n$ (the number of time periods).

If payments are made monthly (that is, 12 times per year), then r is replaced by $\frac{r}{12}$ (the compounding rate) and n is replaced by $12n$ (the number of time periods).

Immediate Annuity - Payments are made at the end of every time period in immediate annuity.

Basic formula for an immediate annuity - The accumulated value A of an immediate annuity for n annual payments of an amount C at an interest rate r per cent per annum, compounded annually, is given by.

$$A = \frac{C}{i}\left[(1+i)^n - 1\right], \text{ where } i = \frac{r}{100}$$

Also, the present value P of such an immediate annuity is given by

$$P = \frac{C}{i}\,[1 - (1 + i)^{-n}]$$

The present value and the future value of an annuity have the following relations.

$$A = P(1 + i)^n,$$

$$\frac{1}{p} - \frac{1}{A} = \frac{i}{C}.$$

SOLVED EXAMPLES

Ex. 1: Find the accumulated value after 3 years of an immediate annuity of Rs. 5000 p.a. with interest compounded at 4% p.a. [given $(1.04)^3 = 1.12490$]

Solution:

The problem states that C = 5000, r = 4% p.a., and n = 3 years.

Then, accumulated value is given by

$$A = \frac{C}{i}\left[(1+i)^n - 1\right] \quad \left\{i = \frac{r}{100} = 0.04\right\}$$

$$= \frac{5000}{0.04}[(1+0.04)^3 - 1]$$

$$= \frac{5000}{0.04}[(1.04)^3 - 1]$$

$$= \frac{5000}{0.04}[1.1249 - 1]$$

$$\therefore \quad A = \frac{5000}{0.04}[1.1249 - 1]$$

$$\therefore \quad A = \frac{5000}{0.04}(0.1249)$$

$$\therefore \quad A = 5000(3.12250)$$

$$\therefore \quad A = 15,612.50$$

The accumulated value of the annuity is Rs. 15,612.50

Ex. 2: A person plans to accumulate a sum of Rs. 5,00,000 in 5 years for higher education of his son. How much should he save every year if he gets interest compounded at 10% p.a.? [Given : $(1.10)^5 = 1.61051$]

Solution: From the problem, we have

A = Rs.5,00,000, r = 10%p.a., and n = 5 years.

$$\therefore \quad i = \frac{r}{100} = \frac{10}{100} = 0.10$$

We find C as follows,

$$A = \frac{C}{i}[(1+i)^n - 1]$$

$$\therefore \quad 5,00,000 = \frac{C}{0.1}[(1+0.1)^5 - 1]$$

$$\therefore \quad 5,00,000 = \frac{C}{0.1}[(1.1)^5 - 1]$$

$$\therefore \quad 5,00.000 = \frac{C}{0.1}[1.61051 - 1]$$

$$\therefore \quad 5,00.000 = \frac{C}{0.1}(0.61051)$$

$$\therefore \quad C = \frac{500000 \times 0.1}{0.61051}$$

$$\therefore \quad C = \frac{500000}{0.61051} = 81898.74$$

That is, the person should save Rs.81898.74 every year for 5 years to get Rs.5,00,000 at the end of 5 years.

Ex. 3: Mr. X saved Rs.5000 every year for some years. At the end of this period, he received an accumulated amount of Rs.23205. Find the number of years if the interest was compounded at 10% p.a. [Given : $(1.1)^4 = 1.4641$]

Solution: From the problem, we have A = Rs.23205, C = Rs.5000 and r = 10% p.a.

$$\therefore \quad i = \frac{r}{100} = \frac{10}{100} = 0.10$$

The value of n is found as follows.

$$A = \frac{C}{i}[(1+i)^n - 1]$$

$$\therefore \quad 23205 = \frac{5000}{0.1}[(1+0.1)^n - 1]$$

$$\therefore \quad \frac{23205 \times 0.1}{5000} = (1.1)^n - 1$$

$$\therefore \quad \frac{2320.5}{5000} = (1.1)^n - 1$$

$$\therefore \quad 0.4641 + 1 = (1.1)^n$$

$$\therefore \quad 1.4641 = (1.1)^n$$

$$\text{since } (1.1)^4 = 1.4641$$

$$\therefore \quad n = 4 \text{ years}$$

Ex. 4: Find the rate of interest compounded annually if an immediate annuity of Rs.20,000 per year amounts to Rs.41,000 in 2 years.

Solution: From the problem, we have A = Rs.41,000, C = Rs.20,000 and n = 2 years. The value of n is then found as follows.

$$A = \frac{C}{i}[(1+i)^n - 1]$$

$$\therefore \quad 41000 = \frac{20000}{i}[(1+i)^2 - 1]$$

$$\therefore \quad \frac{41000}{20000} = \frac{1 + 2i + i^2 - 1}{i}$$

$\therefore \quad 2.05 = \dfrac{2i + i^2}{i}$

$\therefore \quad 2.05 = 2 + i$

$\therefore \quad i = 2.05 - 2$

$\therefore \quad i = 0.05$ but $i = \dfrac{r}{100}$

$\therefore \quad r = i \times 100$

$\therefore \quad r = 5$

$\therefore$ the rate of interest is 5% p.a.

Ex. 5: A person deposited Rs.15,000 every six months for 2 years. The rate of interest is 10% p.a. compounded half yearly. Find the amount accumulated at the end of 2 years.

[Given : $(1.05)^4 = 1.2155$]

Solution:

Since an amount of Rs.15,000 is deposited every six months. It is a case of immediate annuity. The problem states that C = Rs. 15000. Rate of interest is 10% p.a. Therefore, it is 5% for six months. That is, $r = 5\%$.

$\therefore \quad i = \dfrac{r}{100} = \dfrac{5}{100} = 0.05$

The number of half years in 2 years is 4. and therefore $n = 4$.

Now we use the formula for accumulated value A and get

$$A = \frac{C}{i}[(1+i)^n - 1]$$

$$\therefore \quad A = \frac{15000}{0.05}[(1+0.05)^4 - 1]$$

$= 3,00,000\ [1.2155 - 1]$

$= 3,00,000\ [0.2155]$

$= 64650$

$\therefore$ The accumulated amount after 2 years is Rs.64650.

Ex. 6: A person deposits Rs.3000 in a bank every quarter. The interest is 8% compounded every quarter. Find the accumulated amount at the end of 1 year. [*Given* : $(1.02)^4 = 1.0824$]

Solution:

Since the amount is deposited every quarter, it is an immediate annuity. From the given problem. C = 3000. The rate of interest is 8% p.a. and hence it is 2% per quarter. That is, $r = 0.02$ and hence $i = 0.02$. The number of quarters in a year is 4. That is, $n = 4$. We use the formula for accumulated amount to obtain

$$A = \frac{C}{i}[(1+i)^n - 1]$$

$= \dfrac{3000}{0.02}[(1+0.02)^4 - 1]$

$= 1,50,000\ [(1.02)^4 - 1]$

$= 1,50,000\ [1.0824 - 1]$

$= 1,50,000\ [0.0824]$

$= 12360.$

$\therefore$ Accumulated amount after 1 year is Rs.12,360

Ex. 7: Find the present value of an immediate annuity of Rs.50,000 per annum for 4 years with interest compounded at 10% p.a.

[*Given* : $(1.1)^{-4} = 0.6830$]

Solution:

From the problem, we get $C = 50,000$, $n = 4$ years, $r = 10\%$ p.a., so that $i = 0.1$. We use the formula for present value and get

$$P = \frac{C}{i}[1 - (1+i)^{-n}]$$

$$\therefore \quad P = \frac{50000}{0.1}[1 - (1+0.1)^{-4}]$$

$$\therefore \quad P = \frac{50000}{0.1}[1 - (1.10)^{-4}]$$

$= \dfrac{50000}{0.1}[1 - 0.68300]$

$= 500000[0.3170]$

$= 1,58,493.$

$\therefore$ Present value of the given annuity is Rs.1,58,493.

Ex. 8: The present value of an immideiate annuity paid for 4 years with interest accumulated at 10% p.a. is Rs.20,000. What is its accumulated value after 4 years? [*Given* : $(1.1)^4 = 1.4641$]

Solution:

It is given that $P = 20{,}000$. $n = 4$ years, $r = 10\%$ p.a., so that $i = 0.1$. Using the formula for the relation between A and P. we obtain

$$A = P(1+i)^n$$

$$\therefore A = 20{,}000(1+0.1)^4$$

$$= 20{,}000(1.1)^4$$

$$= 20{,}000(1.4641)$$

$$= 29282.$$

$\therefore$ Accumulated value after 4 years is Rs.29,282

Ex. 9: The present and accumulated values of an immediate annuity paid for some years at interest compounded at 10% p.a. are Rs.4,000 and Rs.8,000 respectively. Find the amount of every annuity paid.

Solution:

It is given that P = Rs.4,000, A = Rs.8,000, $r = 10\%$ p.a., so that $i = 0.1$.

We use the following formula for finding C.

$$\frac{1}{p} - \frac{1}{A} = \frac{i}{C}$$

$$\therefore \frac{1}{4000} - \frac{1}{8000} = \frac{0.1}{C}$$

$$\therefore 0.00025 - 0.000125 = \frac{0.1}{C}$$

$$\therefore 0.000125 = \frac{0.1}{C}$$

$$\therefore C = \frac{0.1}{0.000125}$$

$$= \frac{100000}{125}$$

$$= 800.$$

$\therefore$ Every annuity paid is Rs. 800.

Annuity Due

Payments are made at the beginning of every time period in annuity due.

Basic formula for an annuity due

Let C denote the amount paid at the beginning of each of n years and let r denote the rate of interest per cent per annum.

Let $i = \frac{r}{100}$

The accumlated value A' is given by

$$A' = \frac{C(1+i)}{i}[(1+i)^n - 1]$$

The present value P' is given by

$$P' = \frac{C(1+i)}{i}[1 - (1+i)^{-n}]$$

A' and P' have the following relations.

$$A' = P'(1+i)^n$$

$$\frac{1}{P'} - \frac{1}{A'} = \frac{i}{C(1+i)}$$

Note:

We can use the formula of immediate annuity for anuity due only by replacing C by $C(1+i)$

SOLVED EXAMPLES

Ex. 1: Find an accumulated value of an annuity due of Rs 2,000 per annum for 4 years at 10% p.a. [*Given* $(1.1)^4 = 1.4641$]

Solution: Given C = Rs 2,000, $n = 4$ years,

$r = 10\%$ p.a. so that $i = 0.1$

We use the formula for accumulated value A' of an annuity due to get

$$A' = \frac{C(1+i)}{i}[(1+i)^n - 1]$$

$$\therefore A' = \frac{2000(1+0.1)}{0.1}[(1+0.1)^4 - 1]$$

$$\therefore A' = \frac{2000(1.1)}{0.1}[1.4641 - 1]$$

$= 22000\ (0.4641)$

$= 10210.2$

$\therefore$ Accumulated value is Rs 10,210.2

Ex.2: Find the present value of an annuity due of Rs.5,000 to be paid per quarter at 16% p.a. compounded quarterly for 1 year. [Given: $(1.04)^{-4} = 0.8548$]

Solution:

Given C = Rs. 5000. Rate of interest is 16% p.a. and hence it is 4% per quarter. This gives $i = 0.04$. Finally, $n = 4$.

We use the formula for present value of annuity due to obtain

$$P' = \frac{C(1+i)}{i}\,[1 - (1+i)^{-n}]$$

$$\therefore\ P' = \frac{5000(1+0.04)}{0.04}\,[1 - (1+0.04)^{-4}]$$

$$= \frac{5000(1.04)}{0.04}\,[1 - (0.8548)]$$

$= 1,25,000\ (1.04)(0.1452)$

$= 18876$

$\therefore$ Present Value is Rs 18,876

Sinking Fund:

A sinking fund is a fund established by financial organization by setting aside revenue over a period of time compounded annually, to fund a future capital expense, or repayment of a long-term debt.

SOLVED EXAMPLES

Ex. The cost of a machine is Rs.10 lakh and its effective life is 12 years. The scrap realizes only Rs.50,000. What amount should be kept aside at the end of every year to accumulate Rs.9,50,000 at compound interest 5% p.a.? [Given $(1.05)^{12}$ = 1.796]

Solution: Here A = Rs. 10 lakh, r = 5%,

$$i = \frac{r}{100} = 0.05, \text{ and } n = 12.$$

A = 10,00,000 − 50,000

= 9,50,000

$$\text{Now } A = \frac{C}{i}\,[(1+i)^n - 1]$$

$$\therefore\ 9,50,000 = \frac{C}{i}\,[(1+i)^n - 1]$$

$$\therefore\ C = \frac{950000 \times 0.05}{0.796}$$

= 59673.40

$\therefore$ An amount of Rs.59673.40 should be kept aside at the end of every year for 12 years to get Rs.10 lakh at the end of 12 years.

EXERCISE 2.2

1. Find the accumulated (future) value of annuity of Rs.800 for 3 years at interest rate 8% compounded annually.
   [*Given* $(1.08)^3 = 1.2597$]
2. A person invested Rs.5,000 every year in finance company that offered him interest compounded at 10% p.a., what is the amount accumulated after 4 years?
   [*Given* $(1.1)^4 = 1.4641$]
3. Find the amount accumulated after 2 years if a sum of Rs.24,000 is invested every six months at 12% p.a. compounded half yearly. [*Given* $(1.06)^4 = 1.2625$]
4. Find accumulated value after 1 year of an annuity immediate in which Rs.10,000 are invested every quarter at 16% p.a. compounded quarterly.
   [*Given* $(1.04)^4 = 1.1699$]
5. Find the present value of an annuity immediate of Rs.36,000 p.a. for 3 years at 9% p.a. compounded annually.
   [*Given* $(1.09)^{-3} = 0.7722$]
6. Find the present value of an ordinary annuity of Rs.63,000 p.a. for 4 years at 14% p.a. compounded annually.
   [*Given* $(1.14)^{-4} = 0.5921$]

7. A lady plans to save for her daughter's marriage. She wishes to accumulate a sum of Rs.4,64,100 at the end of 4 years. What amount should she invest every year if she gets an interest of 10% p.a. compounded annually? [*Given* $(1.1)^4 = 1.4641$]

8. A person wants to create a fund of Rs.6,96,150 after 4 years at the time of his retirement. He decides to invest a fixed amount at the end of every year in a bank that offers him interest of 10% p.a. compounded annually. What amount should he invest every year? [*Given* $(1.1)^4 = 1.4641$]

9. Find the rate of interest compounded annually if an annuity immediate at Rs.20,000 per year amounts to Rs.2,60,000 in 3 years.

10. Find the number of years for which an annuity of Rs.500 is paid at the end of every year, if the accumulated amount works out to be Rs. 1,655 when interest is compounded annually at 10% p.a.

11. Find the accumulated value of annuity due of Rs.1000 p.a. for 3 years at 10% p.a. compounded annually. [*Given* $(1.1)^3 = 1.331$]

12. A person plans to put Rs.400 at the beginning of each year for 2 years in a deposit that gives interest at 2% p.a. compounded annually. Find the amount that will be accumulated at the end of 2 years.

13. Find the present value of an annuity due of Rs.600 to be paid quarterly at 32% p.a. compounded quarterly. [*Given* $(1.08)^{-4} = 0.7350$]

14. An annuity immediate is to be paid for some years at 12% p.a. The present value of the annuity is Rs.10,000 and the accumulated value is Rs.20,000. Find the amount of each annuity payment.

15. For an annuity immediate paid for 3 years with interest compounded at 10% p.a., the present value is Rs.24,000. What will be the accumulated value after 3 years?

[*Given* $(1.1)^3 = 1.331$]

16. A person sets up a sinking fund in order to have Rs.1,00,000 after 10 years. What amount should be deposited bi-annually in the account that pays him 5% p.a. compounded semi-annually? [*Given* $(1.025)^{20} = 1.675$]

Let's Remember

- Premium is paid on insured value.
- Agent's commission is paid on premium.
- $$\text{Claim} = \frac{\text{PolicyValue}}{\text{PropertyValue}} \times \text{loss}$$
- **Immediate Annuity**

  Amount of accumulated (future) Value = A

  $$A = \frac{C}{i}\left[(1+i)^n - 1\right]$$

  $$\text{Present value } P = \frac{C}{i}\left[1 - (1+i)^{-n}\right]$$

  $$A = P(1+i)^n$$

  $$\frac{1}{P} - \frac{1}{A} = \frac{i}{C}$$

- **Annuity Due**

$$\text{Accumulated value } A' = \frac{C(1+i)}{i}\left[(1+i)^n - 1\right]$$

$$\text{Present value } P' = \frac{C(1+i)}{i}\left[1 - (1+i)^{-n}\right]$$

$$A' = P'(1+i)^n$$

$$\frac{1}{P'} - \frac{1}{A'} = \frac{i}{C(1+i)}$$

MISCELLANEOUS EXERCISE - 2

I) Choose the correct alternative.

1. "A contract that pledges payment of an agreed upon amount to the person (or his/her nominee) on the happening of an event covered against" is technically known as
   a. Death coverage b. Savings for future
   c. Life insurance d. Provident fund

2. Insurance companies collect a fixed amount from their customers at a fixed interval of time. This amount is called
   a. EMI b. Installment
   c. Contribution d. Premium

3. Following are different types of insurance.
   I. Life insurance
   II. Health insurance
   III. Liability insurance
   (a) Only I (b) Only II
   (c) Only III (d) All the three

4. By taking insurance, an individual
   a. Reduces the risk of an accident
   b. Reduces the cost of an accident
   c. Transfers the risk to someone else.
   d. Converts the possibility of large loss to certainty of a small one.

5. You get payments of Rs.8,000 at the beginning of each year for five years at 6%, what is the value of this annuity?
   a. Rs 34,720 b. Rs 39,320
   c. Rs 35,720 d. Rs. 40,000

6. In an ordinary annuity, payments or receipts occur at
   a. Beginning of each period
   b. End of each period
   c. Mid of each period
   d. Quarterly basis

7. Amount of money today which is equal to series of payments in future is called
   a. Normal value of annuity
   b. Sinking value of annuity
   c. Present value of annuity
   d. Future value of annuity

8. Rental payment for an apartment is an example of
   a. Annuity due b. Perpetuity
   c. Ordinary annuity d. Installment

9. ______ is a series of constant cashflows over a limited period of time.
   a. Perpetuity b. Annuity
   c. Present value d. Future value

10. A retirement annuity is particularly attractive to someone who has
    a. A severe illness
    b. Risk of low longevity
    c. Large family
    d. Chance of high longevity

II) Fill in the blanks

1. An installment of money paid for insurance is called __________.
2. General insurance covers all risks except __________.
3. The value of property is called __________.
4. The value of insured property is called __________.
5. The person who receives annuity is called __________.
6. The payment of each single annuity is called __________.
7. The intervening time between payment of two successive installments is called as __________.
8. An annuity where payments continue forever is called __________.

9. If payments of an annuity fall due at the beginning of every period, the series is called annuity ___________.

10. If payments of an annuity fall due at the end of every period, the series is called annuity ___________.

III) State whether each of the following is True or False.

1. General insurance covers life, fire, and theft.
2. The amount of claim cannot exceed the amount of loss.
3. Accident insurance has a period of five years.
4 Premium is the amount paid to the insurance company every month.
5. Payment of every annuity is called an installment.
6. Annuity certain begins on a fixed date and ends when an event happens.
7. Annuity contingent begins and ends on certain fixed dates.
8. The present value of an annuity is the sum of the present value of all installments.
9. The future value of an annuity is the accumulated values of all installments.
10. Sinking fund is set aside at the beginning of a business.

IV) Solve the following problems.

1. A house valued at Rs. 8.00.000 is insured at 75% of its value. If the rate of premium is 0.80 %. Find the premium paid by the owner of the house. If agent's commission is 9% of the premium, find agent's commission.

2. A shopkeeper insures his shop and godown valued at Rs.5,00,000 and Rs.10,00.000 respectively for 80 % of their values. If the rate of premium is 8 %, find the total annual premium.

3 A factory building is insured for $\left(\frac{5}{6}\right)^{th}$ of its value at a rate of premium of 2.50%. If the agent is paid a commission of Rs.2812.50, which is 7.5% of the premium, find the value of the building.

4 A merchant takes fire insurance policy to cover 80 % of the value of his stock. Stock worth Rs.80,000 was completely destroyed in a fire. while the rest of stock was reduced to 20% of its value. If the proportional compensation under the policy was Rs.67,200, find the value of the stock.

5. A 35-year old person takes a policy for Rs.1,00,000 for a period of 20 years. The rate of premium is Rs.76 and the average rate of bonus is Rs.7 per thousand p.a. If he dies after paying 10 annual premiums, what amount will his nominee receive?

6. 15,000 articles costing Rs.200 per dozen were insured against fire for Rs.1,00,000. If 20 % of the articles were burnt completely and 2400 of other articles were damaged to the extent of 80% of their value, find the amount that can be claimed under the policy.

7. For what amount should a cargo worth Rs.25,350 be insured so that in the event of total loss, its value as well as the cost of insurance may be recovered when the rate of premium is 2.5 %.

8. A cargo of grain is insured at $\left(\frac{3}{4}\right)$ % to cover 70% of its value. Rs.1,008 is the amount of premium paid. If the grain is worth Rs. 12 per kg, how many kg of the grain did the cargo contain?

9. 4000 bedsheets worth Rs.6,40,000 were insured for $\left(\frac{3}{7}\right)^{th}$ of their value. Some of the bedsheets were damaged in the rainy season and were reduced to 40% of their value. If the amount recovered against damage was Rs.36,000. find the number of damaged bedsheets.

10. A property valued at Rs.7,00,000 is insured to the extent of Rs.5,60,000 at $\left(\frac{5}{8}\right)$% less 20%. Calculate the saving made in the premium. Find the amount of loss that the owner must bear, including premium, if the property is damaged to the extent of 40 % of its value.

11. Stocks in a shop and godown worth Rs.75,000 and Rs.1,50,000 respectively were insured through an agent who receive 15% of premium as commission. If the shop was insured for 80% and godown for 60% of the value, find the amount of agent's commission when the premium was 0.80% less 20%. If the entire stock in the shop and 20% stock in the godown is destroyed by fire, find the amount that can be claimed under the policy.

12. A person holding a life policy of Rs.1,20,000 for a term of 25 years wants to discontinue after paying premium for 8 years at the rate of Rs.58 per thousand p. a. Find the amount of paid up value he will receive on the policy. Find the amount he will receive if the surrender value granted is 35% of the premiums paid, excluding the first year's premium.

13. A godown valued at Rs.80,000 contained stock worth Rs. 4,80,000. Both were insured against fire. Godown for Rs.50,000 and stock for 80% of its value. A part of stock worth Rs.60,000 was completely destroyed and the rest was reduced to 60% of its value. The amount of damage to the godown is Rs. 40,000. Find the amount that can be claimed under the policy.

14. Find the amount of an ordinary annuity if a payment of Rs. 500 is made at the end of every quarter for 5 years at the rate of 12% per annum compounded quarterly.

15. Find the amount a company should set aside at the end of every year if it wants to buy a machine expected to cost Rs.1,00,000 at the end of 4 years and interest rate is 5% p. a. compounded annually.

16. Find the least number of years for which an annuity of Rs. 3,000 per annum must run in order that its amount exceeds Rs. 60,000 at 10% compounded annually. $[(1.1)^{11} = 2.8531, \ (1.1)^{12} = 3.1384]$

17. Find the rate of interest compounded annually if an ordinary annuity of Rs. 20,000 per year amounts to Rs. 41,000 in 2 years.

18. A person purchases a television by paying Rs.20,000 in cash and promising to pay Rs. 1000 at end of every month for the next 2 years. If money is worth 12% p. a., converted monthly. find the cash price of the television. $[(1.01)^{-24} = 0.7875]$

19. Find the present value of an annuity immediate of Rs. 20,000 per annum for 3 years at 10% p.a. compounded annually.

20. A man borrowed some money and paid back in 3 equal installments of Rs.2160 each. What amount did he borrow if the rate of interest was 20% per annum compounded annually? Also find the total interest charged. $[(1.2)^{-3} = 0.5787]$

21. A company decides to set aside a certain amount at the end of every year to create a sinking fund that should amount to Rs. 9,28,200 in 4 years at 10% p.a. Find the amount to be set aside every year. [$(1.1)^4 = 1.4641$]

22. Find the future value after 2 years if an amount of Rs. 12,000 is invested at the end of every half year at 12% p. a. compounded half yearly. [$(1.06)^4 = 1.2625$]

23. After how many years would an annuity due of Rs. 3.000 p.a. accumulated Rs.19.324.80 at 20% p. a. compounded yearly? [Given $(1.2)^4 = 2.0736$]

24. Some machinery is expected to cost 25% more over its present cost of Rs. 6,96,000 after 20 years. The scrap value of the machinery will realize Rs.1,50,000. What amount should be set aside at the end of every year at 5% p.a. compound interest for 20 years to replace the machinery?

[*Given* $(1.05)^{20} = 2.653$]

Activities

1) Property Value = Rs.1,00,000

Policy value = 70% of property value

$= \boxed{}$

Rate of premium = 0.4%

$$\text{Amount of premium} = \frac{0.4}{100} \times \boxed{}$$

= Rs. 280

Property worth Rs. 60,000 is destroyed

$$\therefore \quad \text{Claim} = \text{loss} \times \frac{\text{PolicyValue}}{\text{PropertyValue}}$$

$$= \boxed{} \times \frac{7}{\boxed{}}$$

= Rs. 42,000

Now, the property worth Rs.60,000 is totally destroyed and in addition the remaining property is so damaged as to reduce its value by 40%

$$\therefore \quad \text{Loss} = 60{,}000 + \frac{40}{100} \times \boxed{}$$

= 60,000 + 16,000

$= \text{Rs.} \boxed{}$

$$\therefore \quad \text{Claim} = \boxed{} \times \frac{7}{10}$$

= Rs. 53,200

2. Policy value = Rs. 70,000

Period of policy = 15 years

Rate of premium = Rs. 56.50 per thousand p.a.

$$\therefore \text{Amount of premium} = \frac{56.50}{1000} \times \boxed{}$$

= Rs.3955

$\therefore$ Total premium paid = 3955 × $\boxed{}$

= Rs.59,325

Rate of bonus = Rs. 6 per thousand p.a.

$\therefore$ Amount of bonus = 6 × $\boxed{}$

= Rs.420

$\therefore$ Bonus for 15 years = 420 × $\boxed{}$

= Rs.6,300

$\therefore$ The person gets Rs. = $\boxed{}$ + 6300

= Rs.76,300

$\therefore$ Benefit = $\boxed{}$ − 59,325

= Rs.16,975.

3. For an immediate annuity,

P = Rs.2000, A = Rs.4000

r = 10% p.a.

$$\therefore \quad i = \frac{r}{100} = \frac{\boxed{}}{100} = 0.1$$

$\therefore \quad \frac{1}{P} - \frac{1}{A} = \frac{i}{C}$

$\therefore \quad \frac{1}{\square} - \frac{1}{\square} = \frac{0.1}{C}$

$\therefore \quad \frac{\square}{4000} = \frac{0.1}{C}$

$\therefore \quad C = \text{Rs.}\ \square$

4. For an annuity due,

C = Rs.2000, rate = 16% p.a. compounded quarterly for 1 year

$\therefore$ Rate of interest per quarter $= \frac{\square}{4}$

$= 4$

$\therefore \quad r = 4\%$

$\therefore \quad i = \frac{r}{100} = \frac{\square}{100} = 0.04$

n = Number of quarters

$= 4 \times 1$

$= \square$

$\therefore \quad P' = \frac{C(1+i)}{i}\left[1-(1+i)^{-n}\right]$

$\therefore P' = \frac{\square(1+\square)}{0.04}\left[1-(\square+0.04)^{-\square}\right]$

$= \frac{2000(\square)}{\square}\left[1-(\square)^{-4}\right]$

$= 50{,}000\ (\square)\ (1 - 0.8548)$

$= 50{,}000\ \ (1.04)\ (\square)$

= Rs.7,550.40

5. The cost of machinery = Rs.10,00,000

Effective life of machinery = 12 years

Scrap value of machinery = Rs. 50,000

$r = 5\%$ p.a.

$\therefore \quad i = \frac{r}{100} = \frac{\square}{\square} = 0.05$

$A = 10{,}00{,}000 - 50{,}000$

$= \square$

For an immediate annuity

$A = \frac{C}{i}\left[(1+i)^n - 1\right]$

$\therefore \quad \square = \frac{C}{\square}\left[(1+0.05)^{\square} - 1\right]$

$\therefore\ 9{,}50{,}000 = \frac{C}{0.05}\left[1.797 - \square\right]$

$\therefore \quad C = \frac{950000 \times \square}{0.797}$

= Rs.59,598.40

✦✦✦